D'Alembert's Principle

→ It is the application of Newton's second law of motion.

→ According to Newton's 2nd law, the rate of change of momentum is directly proportional to the applied force & takes place in the direction of force of action. i.e. force is directly proportional to product of mass & acceleration. i.e. $F = m \cdot a$.

→ Instead of single force, if a system of forces act upon a particle, the statement reduces to "Resultant force is equal to the product of mass & acceleration in the direction of the resultant force.

i.e. $R = m \cdot a$.

where R is the resultant force acting on the particle.

→ Hence many times Newton's 2nd law can be stated as "A particle acted upon by an unbalanced system of forces has an acceleration directly proportional & is inline with resultant force.

→ The above eqn seen in a different angle that $R - ma = 0$.

where "$-ma$" may be considered as force of magnitude "ma", applied in the opposite direction of motion & is termed as the inertia force or reverse effective force.

→ "The system of forces acting on a moving body is in dynamic equlibrium with the inertia force of the body". This statement is known as D'Alembert's Principle.

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Fig ① subjected to a system of forces causing the body to move with an acceleration "a" in the direction of resultant.

Then apply a force equal to "ma" in the reverse direction of acceleration as per ②.

Now as per D'Alembert's principle, the eqn of equilibrium $\Sigma F_x = 0$ & $\Sigma F_y = 0$ may be used for the system of force.

Ex-1 Calculate the magnitude & ~~direction~~ acceleration of each of the bodies.

(a)

Sum of forces in x-direction = mass x acceleration in x direction

$\Rightarrow \Sigma F_x = ma_x \Rightarrow 5 = (1)\, a_x$

$\Rightarrow a_x = 5\ m/s^2$

Similarly, sum of forces in y-direction = mass x acc. in y-dir.

$\Rightarrow \Sigma F_y = ma_y \Rightarrow 6 = (1)\, a_y \Rightarrow a_y = 6\ m/s^2$

Resultant acceleration $= a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 6^2} = \underline{7.81\ m/s^2}$

(b)

$\Sigma F_x = ma_x \Rightarrow 15 - 12 = 5 \times a_x$

$\Rightarrow a_x = 3/5\ m/s^2$

$\Sigma F_y = ma_y \Rightarrow 45 - 10 = 5 \times a_y$

$\Rightarrow a_y = 7\ m/s^2$

$\Rightarrow a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(\frac{3}{5}\right)^2 + 7^2} = \underline{7.02\ m/s^2}$

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(c)

$\Sigma f_x = m.a_x$

$\Rightarrow 45 + 25\cos 60° = 20\, a_x$

$\Rightarrow a_x = 2.875\ m/s^2$

$\Sigma f_y = m a_y$

$\Rightarrow -25 \sin 60° = 20\, a_y$

$\Rightarrow a_y = -1.0825\ m/s^2$

$\therefore a = \sqrt{a_x^2 + a_y^2} = \underline{3.071\ m/s^2}$

Ex-2

A vehicle of mass 2000 kg ~~direc~~ driven up, ~~on~~ a 10° slope by a tractive force of 3kN. Air resistance is 450N. Neglecting Road resistance Calculate the acceleration of the vehicle.

Ans

→ Air resistance is against the motion, so shown in opposite direction of motion.

→ Neglect road resistance, so no frictional force.

Weight of vehicle $= 2000 \times 9.81 = \underline{19620\ N}$

$\Sigma f_x = m a$

$\Rightarrow 3000 - 450 - 19620 \sin 10° = 2000\, a$

$\Rightarrow \boxed{a = -0.428\ m/s^2}$

Ex-3 A man jumps off a railway Platform with a block of mass m kg in his hand. Find the thrust of the block on his hand.

Ans Free body diagram of the block.

$\Sigma f_y = m.a$

$R - mg = m.(-g)$

$\Rightarrow R = 0$

Thus no thrust exerted on hand.

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Ex-4 Block A & cylinder B, as shown in the figure move horizontally with an acceleration of $2.5\ m/s^2$. Find the force exerted on the cylinder.

Ans

R_1 & R_2 are the reaction forces exerted by the block A on the cylinder B.

For equilibrium in Y-direction

$$\Sigma F_y = 0$$

$$\Rightarrow R_2 + R_1 \cos 30^\circ - W = 0$$

$$\Rightarrow R_2 + R_1 \cos 30 - 0.5 \times 9.81 = 0$$

equilibrium in x-direction

$$\Sigma F_x = ma$$

$$R_1 \sin 30^\circ = 0.5 \times 2.5$$

$$\Rightarrow \underline{R_1 = 2.5\ N}$$

$$\therefore R_2 + 2.5 \cos 30^\circ - 0.5 \times 9.81 = 0$$

$$\underline{R_2 = 2.74\ N}$$

Ex-5 A bullet of 0.5 kg strikes an wooden Plank of 250 mm thick with a velocity of 240 m/s. & comes out on the other side with velocity of 120 m/s. Find the (a) Resistance faced by bullet (b) what will be the thickness of Planck to reduce the velocity to zero.

Ans Given $x = 250\ mm = 0.25\ m$, $u = 240\ m/s$

$v = 120\ m/s$

$$v^2 = u^2 + 2ax \Rightarrow 120^2 = 240^2 + 2 \times a \times 0.25$$

$$\Rightarrow a = -86400\ m/s^2$$

Resistance faced by bullet.

$$F = ma \Rightarrow F = 0.5 \times (-86400)$$

$$\Rightarrow F = -43200\ N$$

To bring velocity to zero, thickness of Planck ?

$\therefore v = 0$, $u = 240\ m/s$, $a = -86400\ m/s^2 \Rightarrow v^2 = u^2 + 2ax$

$$\Rightarrow 0 = 240^2 - 2 \times 86400 \times x \Rightarrow \underline{x = 0.333\ m}$$

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Ex-6 Blocks are arranged as shown in the figure. Find the horizontal force applied at A to cause a motion to start at B.

Block B

$\Sigma F_y = 0 \Rightarrow R_1 - 78.48\cos 10° = 0$

$\Rightarrow R_1 = 77.28\ N$

$\Sigma F_x = ma$

$0.25 R_1 - 78.48 \sin 10° = 8 \times a$

$\Rightarrow 0.25 \times 77.28 - 78.48 \sin 10 = 8a$

$\Rightarrow a = 0.711\ m/s^2$

Block A

$\Sigma F_x = 0 \Rightarrow -196.2\cos 10 - R_2 - R_1 - F\sin 10° = 0$

$\Rightarrow R_2 = 270.5 + F\sin 10°$

$\Sigma F_x = ma$

$F\cos 10° - 196.2\sin 10° - 0.25 R_1 - 0.35 R_2 = 20a$

$\Rightarrow F\cos 10° - 196.2\sin 10° - 0.25(77.28) - 0.35(270.5 + F\sin 10°) = 20(0.711)$

$\Rightarrow \boxed{F = 175.62\ N}$ Ans

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Ex-7 Two blocks, m_1 of mass 2 kg & m_2 of mass 4.5 kg are initially at rest. They are connected by a light inextensible rope which passes over a pulley. Assume the pulley to be massless & frictionless. Determine the acceleration of each block when the force applied to the pulley is (a) $F = 15N$ (b) $F = 76N$. Also find the force required to lift both the block.

Ans

$m_1 = 2kg \Rightarrow W_1 = 2 \times 9.81 = 19.62 N$

$m_2 = 4.5 kg \Rightarrow W_2 = 4.5 \times 9.81 = 44.145 N$

(a) when $F = 15 N$

$\Rightarrow$ Tension in each rope will be 7.5 N (neglecting weight of pulley)

$\therefore$ The force is insufficient to lift either of the masses.

$\therefore a_1 = a_2 = 0$, $a_{pulley} = 0$

(b) when $F = 76 N$

$\Rightarrow$ Tension in each rope $= \frac{76}{2} = 38 N$

$\therefore$ m_2 will not lift as $T_2 < mg$ $\therefore a_2 = 0$

but m_1 will lift as $T_1 > mg$

$\therefore \Sigma F_y = ma$

$\Rightarrow 38 - 19.62 = 2 \times a$

$\Rightarrow a = 9.19 m/s^2$

(c) To lift the m_2, the tension in rope must be greater than or equal to $m_2 g$

i.e. $T_2 \geq m_2 g = 44.145 N$

$\therefore T = 2 \times 44.145 = 88.29 N$

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Ex.8 Block A of 150 kg & Block B of 50 kg are connected by an inextensible cable which passes over pulleys as shown in the figure. Assume pulleys to be massless & frictionless. Determine the tension in rope & acceleration of the blocks.

Top surface as reference.

$$x_C + x_C - x_D + x_B - x_D = \text{cons.}$$

$$\Rightarrow 2x_C + x_B = \text{cons.}$$

$$\Rightarrow 2x_A + x_B = \text{cons}$$

& $x_D = \text{cons.}$

$$\therefore 2x_A + x_B = \text{cons.}$$

Differentiating twice, we get

$$2a_A + a_B = 0 \Rightarrow \boxed{a_B = -2a_A}$$

Pulley C FBD $\Sigma F_Y = 0 \Rightarrow 2T - T_1 = 0 \Rightarrow \boxed{T_1 = 2T}$

Block A $\Sigma F_Y = ma \Rightarrow T_1 - 1471.5 = -150 \times a_A$

$$\Rightarrow -1471 + 2T = -150a_A \rightarrow (1)$$

Block B $\Sigma F_Y = ma \Rightarrow T - 490.5 = -50\, a_B$

$$\Rightarrow T - 490.5 = 100a_A \quad -(2)$$

$$(\because a_B = -2a_A)$$

eqn (1) − eqn (2) × 2 gives

$$\{-1471.5 + 2(100a_A + 490.5) + 150a_A\} - (2T - 490.5 - 100a_A) = 0$$

$$\Rightarrow a_A = 1.4\ m/s^2, \quad a_B = -2.8\ m/s^2$$

$$\therefore T = 630.5\ N \quad \text{Ans}$$

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Pt-9

① mass m will just support mass M.
② Mass m is now added to mass M.

When mass "m" supports "M", the system is in static equilibrium

Pulley B, $\Sigma F_y = 0$ $\boxed{T_1 = 2T_2}$

mass m, $\Sigma F_y = 0$, $\boxed{T_2 = mg}$

mass M, $\Sigma F_y = 0$, $\boxed{T_1 + T_2 = Mg}$

$\Rightarrow 3T_2 = Mg$

$\Rightarrow 3mg = Mg \Rightarrow \boxed{M = 3m}$

Additional mass "m" added to M.

On addition of mass "m", D will start moving downward.

Top surface taken as reference.

$x_D - x_A + x_B - x_A + x_D - x_B + x_C - x_B = \text{cons}$

$\Rightarrow 2x_D - x_B + x_C = \text{cons}$ — ①

$\Rightarrow x_D - x_A + x_B - x_A = \text{cons}$

$\Rightarrow x_D + x_B = \text{cons}$ — ⅡⅠ

① + ⅡⅠ $\Rightarrow 3x_D + x_C = \text{cons}$

Twice Differentiate we get

$\boxed{3a_D + a_C = 0}$

Block D $\Sigma F_y = ma$

$T_1 + T_2 - (M+m)g = -(M+m)a_D$

$4mg - 3T_2 = 4ma_D$ $(\because M = 3m, T_1 = 2T_2)$ — ⅡⅠ

Block C $\Sigma F_y = ma$, $T_2 - mg = -ma_C$ — Ⅳ

ⅡⅠ + 3×3 gives $mg = 4ma_D - 3ma_C$

Put $a_C = -3a_D$

$a_D = g/13$, $a_C = -3(g/13)$

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Ex-10 A system of 3 moveable pulleys employed to raise a block of mass 115 kg by a 50 kg block. Assume no friction & no masses for pulleys. Find acceleration of mass 115 kg block.

Ans

50 kg block move down by "x"

Pulley ① moves up by $x/2$

Pulley ② moves up by $x/4$

Pulley ③ moves up by $x/8$

So 50 kg mass block has downward acceleration of $8a$ & 115 kg block have an upward acceleration of a.

We know. $2T_2 = T_1$

& $T_3 = T_2$; $T_3 = T_1/4$

$\therefore \Sigma F_y = ma$

$\Rightarrow T_3 = 50 \times 9.81 = -50 \times 8a$ — ①

115 kg block

$2T_1 - 115 \times 9.81 = 115a$ — ②

Putting value of T_3 in ①

$T_1 - 200 \times 9.81 = -200(8a)$ — ③

$\therefore$ ② − 2×③ $a = 0.8433\ m/s^2$ **Ans**

Ex-11 Two blocks are connected by string. The block of 28 kg lies on a rough plane of slope 40° & block of 20 kg hangs freely. Coefficient of friction b/w plane & block is 0.2. 28 kg block moving down the plane. Find acceleration of masses.

Block A $\Sigma F_x = ma$, ~~$T + \mu R - 28 \times 9.81 \times \sin 40°$~~

$T + \mu R - 28 \times 9.81 \times \sin 40° = -28a$

$\Sigma F_y = 0$, $R - 28 \times 9.81 \times \cos 40° = 0$

Substitute R & μ, $T + 0.2 \times 28 \times 9.81 \times \cos 40° - 28 \times 9.81 \times \sin 40° = -28a$

$\Rightarrow T + 28a = 134.48$ — ①

Block-B $\Sigma F_y = ma$, $T - 20 \times 9.81 = 20a \Rightarrow T - 20a = 196$ — ②

① − ②, $48a = -61.72$

$\Rightarrow a = -1.286\ m/s^2$ **Ans**

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Ex-12 A 480 kN engine pulls a 1500 kN carriage up an incline of 1 in 120. Engine starts from rest with an uniform acceleration against a resistance of 4 N/kN. Max^m speed of 36 km/hr reached in a distance of 1 km. Calculate tension in coupling connecting carriage & tractive force given by engine. (25)

Ans

Resistance to motion = 4 N/kN

f = Total resistance = $(480+1500)4 = 7920\,N$.

Given, $u = 0$, $v = 36\,km/hr = 10\,m/s$, $x_0 = 0$, $x = 1000\,m$.

$v^2 = u^2 + 2ax$

$10^2 = 2 \times a \times 1000 \Rightarrow a = 0.05\,m/s^2$

Engine + carriage

$\Sigma F_x = ma$

$$P - F - (480+1500)\times 10^3 \times \sin\theta = \frac{(480+1500)\times 10^3}{9.81} \times 0.05$$

$\sin\theta \simeq \tan\theta$ for small values of θ i.e. $1/120$

Put $\sin\theta = \frac{1}{120}$

$P = 34511.74\,N$

Engine

T = Tension in coupling

F_1 = Resistance to motion.

$F_1 = 480 \times 4 = 1920\,N$

$\Sigma F_x = ma$

$$P - T - F_1 - 480\times 10^3 \sin\theta = \frac{480\times 10^3}{9.81} \times 0.05$$

$$34511.74 - T - 1920 - 480\times 10^3 \times \frac{1}{120} = \frac{480\times 10^3}{9.81} \times 0.05$$

$T = 26.145\,N$ Ans

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